Part 4

Symbolic Partial Order Reduction

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Partial Order Reduction

Partial Order Reduction uses independence to reduce the state space for model checking, while still detecting all errors. [Clarke, Grumberg, Peled 1999]

- Full search examines each enabled transition at each state.
- Partial Order Reduction examines an ample set of transitions at each state.
- All linear time temporal properties not involving next-time are preserved
Definitions about Transitions

For a transition $a$ we write

$$en_a(S) : States \rightarrow Boolean$$

for the enabling condition at state $S$, and

$$a(S) : States \rightarrow States$$

for the successor state from state $S$ by transition $a$. 
Independent Transitions

(a and b both enabled)

Transitions $a, b$ are independent iff for all states $S'$:

- $en_a(S) \land en_b(S) \Rightarrow en_a(b(S)) \land en_b(a(S))$
- $en_a(S) \land en_b(S) \Rightarrow a(b(S)) = b(a(S))$

A traditional implementation uses variable occurrence to check independence.

This approach is not accurate enough for protocols having complex transition rules.
Symbolic Partial Order Reduction

- We are investigating the use of symbolic simulation and propositional reasoning to implement a more accurate check for independence.
- Some initial results are presented in a technical report

A Symbolic Partial Order Reduction Algorithm for Rule Based Transition Systems,
R. Bhattacharya, S. German, G. Goplakrishnan
UUCS-03-028, School of Computing, University of Utah, December 2003
An Example

- Token-passing mutual exclusion protocol
- Two concentric rings
- Example has characteristics of a cache protocol.
Protocol Rules

- If a node does not have the token, it can issue a request by sending two messages on the ring.

- Requests are either granted or rejected.

- If a granted request returns to the issuing node, the node enters the exclusive state.

- When both messages have returned to issuing node, if the node has received a grant, it will grant requests from other nodes.

Message Status

A message can be in one of the following states:

- Request
- Reject
- Grant
Rules for Granting and Rejecting

1. If a Request arrives at a node in the Granting state, the request is Granted.

2. If a Request arrives at a node that has the Exclusive token but is not ready to Grant, the request is Rejected.

3. If a Request arrives at a node that contains a Grant message for a different owner, the Request is Rejected.

4. If a Request arrives at a node containing a Rejected Request from the same owner, the Request is Rejected.
Message Priority

- A message with status Grant has priority over other messages for transfer out of the node.
Acceleration of Rejected Messages

- Rejected messages have little interaction with system.
- Intuitively, rejected messages can be accelerated.

Acceleration of rejected messages gives 5 to 6 times reduction of state space.
Using Deduction to Justify an Ample Set

If a transition \( t \) is independent of all other transitions, if \( t \) is \textit{invisible} and if we observe a \textit{cycle condition}, then we can use \( \{t\} \) as a ample set.

We would like to show that transfer of Rejected Requests satisfies these conditions:

- Independence is non-trivial because of message priority.
  - Use symbolic checking for independence.

- Independence may only be true in the reachable state space.
  - Prove invariants of the transition system to justify independence.

- It may be necessary to \textit{split} a transition into smaller cases which are independent.

\[
\text{guard} \implies \begin{cases} 
\text{if b then S1 else S2 endif} \\
\text{guard & b} \implies S1 \\
\text{guard & !b} \implies S2 
\end{cases}
\]
Appendix B. Mutual Exclusion Protocol Model

/* ring.m == tutorial example of a ring fabric for a mutual exclusion protocol, showing a manual partial order reduction

   S.M. German   19 August 2004

*/

const node_count: 4;

const reduce: false;  -- set this constant to true to apply PO reduction

type
   node_id: 0 .. node_count - 1;

message_id: 0 .. (2 * node_count) - 1;

active_count: 0 .. node_count - 1;

status_type: enum{inactive, request, reject, grant};

message_type:
   record
      loc: node_id;
      status: status_type;
      endrecord;

node_type:
   record
      active: active_count;
      endrecord;
exclusive: boolean;
requesting: boolean;
endrecord;

var
node: array[node_id] of node_type;
msg: array[message_id] of message_type;

function left(n:node_id): message_id;
begin
  return 2 * n;
end;

function right(n:node_id): message_id;
begin
  return (2 * n) + 1;
end;

function left_succ(n: node_id): node_id;
begin
  return (n + 1) % node_count;
end;

function right_succ(n: node_id): node_id;
begin
  return (n + node_count - 1) % node_count;
end;

function next_node(m: message_id): node_id;
begin
  alias n: msg[m].loc do
    if m % 2 = 0 then return left_succ(n) else return right_succ(n); endif;
  endalias;
end;
function other_message(m: message_id): message_id;
begin
  if m % 2 = 0 then return m + 1 else return m - 1; endif;
end;

function home_node(m: message_id): node_id;
begin
  return m / 2;
end;

function message_priority(m: message_id): boolean;
begin
  alias ms: msg[m] do
  return ms.status = grant
  | !exists mm: message_id do
    msg[mm].loc = ms.loc
    & msg[mm].status = grant
    endexists;
endalias; end;

procedure transfer(m: message_id);
var loc: node_id;
begin
  loc := next_node(m);
  msg[m].loc := loc;
  if loc = home_node(m)
  then
    if msg[m].status = grant then node[loc].exclusive := true; endif;
    clear msg[m];
    if msg[other_message(m)].status = inactive then
      node[loc].requesting := false;
    endif;
  elsif msg[m].status = request
node[loc].exclusive

-- 16 Aug 04
-- loc only grants when both of its messages have returned
  & !node[loc].requesting
  then  msg[m].status := grant;
         node[loc].exclusive := false;

-- 16 Aug 04
-- loc rejects a request if it has the exclusive state but it still has
-- a request message outstanding
  elsif msg[m].status = request
    & ( ( exists mm: message_id do
          msg[mm].loc = loc
          & home_node(m) != home_node(mm)
          & msg[mm].status = grant
          endexists )
     | node[loc].exclusive )
    then  msg[m].status := reject;
  endif;

-- this part is optional:
-- if both messages from a given home are located at the same node on
-- the ring, and one of the messages is rejected, mark the other
-- message as rejected.
  if  msg[other_message(m)].loc = loc
     & ( (msg[m].status = reject & msg[other_message(m)].status = request)
        | (msg[m].status = request & msg[other_message(m)].status = reject)
        then  msg[m].status := reject;
              msg[other_message(m)].status := reject;
     endif;
  end;

startstate
  clear node;
clear msg;
node[0].exclusive := true;
end;

ruleset n: node_id do
  rule "node issues request"
    !node[n].requesting
    & !node[n].exclusive ==>
    begin
      msg[left(n)].status := request;
      msg[right(n)].status := request;
      msg[left(n)].loc := n;
      msg[right(n)].loc := n;
      node[n].requesting := true;
    end; endruleset;

  -- unreduced form of rule
ruleset m: message_id do
  rule "transfer message"
    !reduce
    & msg[m].status != inactive
    & message_priority(m) =>
      transfer(m)
  endrule; endruleset;

ruleset m: message_id do
  rule "transfer unrejected message"
    reduce
    & msg[m].status != inactive
    & message_priority(m)
    & !exists o: message_id do
      msg[o].status = reject
      & message_priority(o)
    endexists
    =>
transfer(m)
endrule; endruleset;

ruleset m: message_id do
    rule "transfer rejected message"
        reduce
        & msg[m].status = reject
        & message_priority(m)
        & !exists o: message_id do
            o < m
            & msg[o].status = reject
            & message_priority(o)
        endexists
        ==> transfer(m)
endrule; endruleset;

invariant "excl" forall i: node_id; j: node_id do
    ! (i != j & node[i].exclusive & node[j].exclusive) endforall;

invariant "exclusive exists"
    exists n: node_id do node[n].exclusive endexists
    | exists m: message_id do msg[m].status = grant endexists;

/* PO reduction data:
8/16 model change: grant is not allowed if request is still active

4 nodes: before 8/16 change, unreduced 30.1M, reduced 5.2M
  30.1 / 5.2 = 5.8

with 8/16 change, unreduced 20.4M, reduced 3.9M
  20.4 / 3.9 = 5.2
*/